Methods for the Design and Analysis of Jet-Flapped Airfoils

N. D. Halsey*
Douglas Aircraft Company, Long Beach, Calif.

Methods for solving both the direct and inverse jet flap airfoil potential flow problems are described. The direct airfoil analysis method is a completely nonlinear iterative method which is applicable to either thick or thin airfoils of arbitrary shape. The very general surface singularity formulation has been extended to include multielement airfoils, ground effects, nonuniform freestreams, inlet flows, jet entrainment effects, etc. Comparisons are given with the results of previous linear and nonlinear methods as well as with experimental data. The inverse (design) method is a more approximate method in which camber and thickness distributions are designed separately. Section shapes are shown for several airfoils designed to have only very small regions of adverse pressure gradient.

Nomenclature

c = length of airfoil chord
c_p = coefficient of pressure
c_l = coefficient of lift
c_μ = coefficient of jet momentum
h = height of airfoil leading edge above ground plane
R = radius of curvature of the jet sheet
s = coordinate along the jet sheet
t = airfoil thickness
V = local flow speed
$\bar{V}$ = average flow speed across a vortex sheet
V_j = jet flow speed
V_n = component of velocity normal to a surface
V = freestream flow speed
x = coordinate parallel to the freestream
y = coordinate perpendicular to the freestream
δ_j = jet deflection angle at the trailing edge relative to the airfoil chord line
γ = strength of a vortex sheet
θ = local angle of inclination of the jet sheet relative to the freestream
ϕ = velocity potential

Introduction

THE possibility of integrating aircraft propulsive and lift systems has been recognized for many years. As long ago as 1938, Hagedorn and Ruden[1] demonstrated that a thin jet sheet deflected from an airfoil's trailing edge could be used in place of a mechanical flap. The jet flap, as the arrangement is called, offers a number of advantages in both low and high-speed flight regimes, most of them a consequence of the more nearly uniform loading over the chord of the airfoil.

Balanced against the aerodynamic advantages of the jet flap are serious practical disadvantages. The additional weight, complexity, and expense of the internal ducting system have stimulated the development of alternative systems, such as the externally-blown flap and upper-surface blown flap, which sacrifice some of the aerodynamic efficiency for the sake of simplicity.

In conducting the tradeoff studies between system efficiency and practicality, airfoil sections which take maximum advantage of the aerodynamic advantages of jet flaps should be used. To date, however, jet-flapped airfoils have basically just been conventional airfoils to which jets have been added. These sections have differed from the optimum sections by unknown amounts.

This paper describes some methods developed over the last couple of years for analyzing the flow over jet-flapped airfoils of given shape and for designing section shapes to obtain given velocity distributions. Using these methods, it should be possible to select the optimum airfoil shape for any particular condition. While these methods are currently only applicable to the internally-ducted jet, the future development of similar methods for the other jet-lift schemes should make possible a more rational basis for choosing among the various systems.

Formulation of the Jet Flap Problem

The methods described below are basically potential flow methods. However, boundary-layer effects on the airfoil surfaces can be considered in the same manner as in conventional airfoil cases, and the effects of entrainment of the freestream into the jet (also a viscous effect) on the flow outside the jet can also be determined. Cases with substantial regions of separated flow are not considered.

Figure 1 briefly summarizes the mathematical formulation of the jet flap problem. The basic field equation is Laplace's ($\nabla^2\phi = 0$), where the velocity at any point is the gradient of the potential ϕ. On all airfoil and jet surfaces the normal component of the velocity is either zero or some other previously specified value (V_n = specified). This is referred to as the kinematic boundary condition in the following discussion.

In addition, the jet satisfies a dynamic boundary condition relating the centrifugal and pressure forces at each point on the jet. If the jet is relatively thin compared to its radius of curvature, it can be shown (see Ref. 2, for example) that in nondimensional terms, the dynamic boundary condition is given by

$$\Delta c_p = c_\mu/(R/c) \tag{1}$$

The usual assumption is made that the jet thickness approaches zero in such a way that the mass flow rate is zero, but the jet momentum coefficient is finite. The jet sheet is then represented by a continuous vortex sheet, with its strength given by Eq. (2).

$$2(\gamma/V_\infty)(\bar{V}/V_\infty) = c_\mu/(R/c) \tag{2}$$

The equivalent of the airfoil Kutta condition is satisfied by requiring the jet vortex strength to go to zero infinitely far behind the airfoil.

Presented as Paper 74-188 at the AIAA 12th Aerospace Sciences Meeting, Washington, D.C., January 30–February 1, 1974; submitted February 11, 1974; revision received May 28, 1974. This research was conducted under the McDonnell Douglas Independent Research and Development Program.

Index categories: Aircraft Aerodynamics (Including Component Aerodynamics); Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Engineer/Scientist, V/STOL Systems, Aerodynamic Technology.

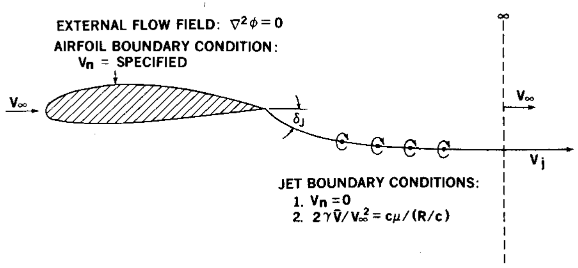

Fig. 1 Mathematical model of the jet flap problem.

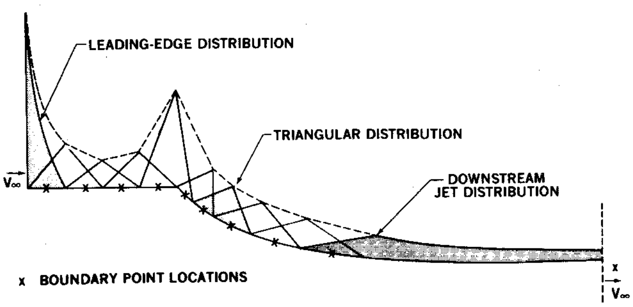

Fig. 2 Finite-element representation of a thin jet-flapped airfoil.

Solution of the Jet Flap Problem—Airfoil Analysis Methods

The difficulty in solving jet flap potential flow problems stems mainly from the unknown shape of the jet sheet. The simplest way around this difficulty is to linearize the problem and satisfy the boundary conditions along a semi-infinite line. The results of linear methods obtained by Spence[2] and others are often surprisingly accurate, at least for the overall airfoil force and moment coefficient calculations. Three-dimensional jet-wing methods based on the same assumptions have also been very successful.[3]

There are many cases, however, in which the small-angle assumptions of the linear theories are clearly violated and others in which more detailed knowledge of the flow about more complex configurations is desired. To analyze these cases and also to assess the limits of applicability of the linear jet flap theories, a nonlinear method making no small-angle assumptions or approximations to the jet shape is needed. Several investigators have attempted to develop such a method, but, as will be discussed later, most are subject to serious criticism.

Of the various methods of solving potential flow problems without making small-angle assumptions, those making use of conformal mapping are generally the most accurate. However, the surface singularity approach can be nearly as accurate in many cases and is much more readily adapted to complicated configurations—like those involving multielement airfoils, ground effects, flow through the surface, etc.

The basic idea behind this approach is that the flowfield produced by any distribution of singularities (sources, vortices, etc.) automatically satisfies Laplace's equation. Since the velocity at any point in the flowfield depends only on the geometry and the singularity strengths, the boundary conditions can be expressed as equations in which the singularity strengths are the only unknown terms. Representing the airfoil geometry and the singularity distributions by a finite number of elements and requiring the boundary conditions to hold at one point in each element reduces the problem to the solution of a set of simultaneous algebraic equations. The kinematic boundary conditions result in linear equations, but the dynamic boundary conditions are nonlinear. Because of this nonlinearity and the unknown jet shape, solution of the jet-flapped airfoil analysis problem inevitably involves an iterative process.

Since many of the calculations in a jet flap potential flow theory are identical with those in a conventional potential flow theory (such as calculation of forces from pressures, solution of matrices, etc.) only the distinctive features of the jet flap method will be discussed. These include the general categories of the general form of the singularity distributions, the jet iteration scheme, and the representation of jet entrainment.

Singularity Distributions

Successful conventional airfoil potential flow methods have been developed utilizing source distributions as the primary singularities[4]; others[5,6] use mainly vortices. For the present application, the vortex approach is adopted, since it has already been established that the thin jet sheet behaves like a vortex sheet. In some cases, sources or sinks are also needed, for example, when the normal component of the velocity at the airfoil surface is nonzero or when there is entrainment into the jet. But in the present formulation the strengths of the source/sink distributions are specified, so that the vortex strengths remain the only unknown terms.

Of the many possible types of vortex distributions, the discrete vortex seems to be the most common in previous nonlinear jet flap methods. The methods developed by Davenport[6] and Herold,[7] which consider only infinitely thin airfoils, use discrete vortices to represent both airfoil and jet. The thick-airfoil method of Sato[8] uses them for the jet only. Although discrete vortex solutions of conventional airfoil problems in some cases can be very successful (most notably the flat plate and the airfoil with a parabolic camberline), in many other cases they are not. Unpublished work at Douglas indicates that the results of cases having discontinuities in the camber angle (jet or flap hinges) are strongly dependent on the number and distribution of vortices used. Other work has shown that, even for the flat plate case in which the correct lift and moment coefficients are obtained, the chordwise loading does not converge to the correct result as the number of vortices is increased. For jet flap problems, where large discontinuities in flow angles are often encountered, it is thus desirable to use more sophisticated vortex distributions. Higher-order formulations also have the further advantage of eliminating the necessity of truncating the jet a finite distance from the trailing edge.

One previous higher-order formulation is described in Ref. 9. This method uses a series expansion for the vortex distribution, which assumes the correct asymptotic behavior at the leading edge and infinitely far downstream on the jet. Unfortunately, however, the series contains far too few terms (only three on the airfoil) to adequately describe the vorticity of arbitrarily shaped airfoils and is probably inadequate for the simple flatplate case as well.

Figure 2 illustrates many of the features of the present approach. Like some earlier linearized jet flap methods[3] and conventional airfoil potential flow methods,[4-6] the geometry is represented by a large number of straight line segments and the vorticity variation over each segment is assumed to be linear (represented by overlapping triangular distributions). At the leading edge of infinitely thin airfoils a square-root singularity is assumed. The jet element farthest downstream is infinitely long and over its length the vorticity decays to zero in the simplest manner having a finite amount of total vorticity and having zero influence at infinity. Airfoils having nonzero thickness are represented in a similar manner, except that the overlapping triangular distributions extend around the entire perimeter of the airfoil.

In addition to the three basic vortex distributions, cases

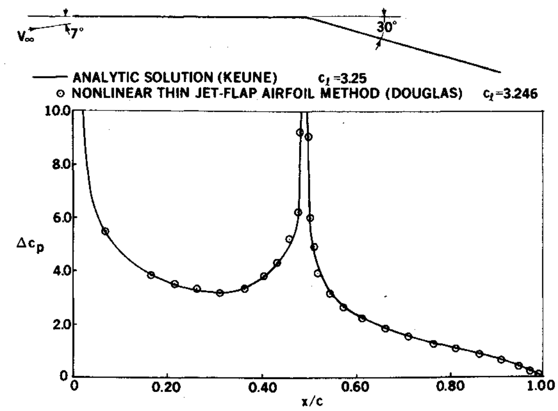

Fig. 3 Accuracy of the present method for a thin airfoil case.

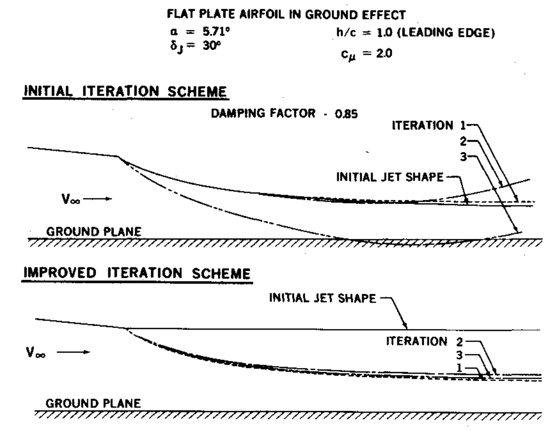

Fig. 5 Comparison of jet iteration schemes.

in which flow is allowed through some surfaces require the use of source distributions. For these cases, the source strength is assumed to be constant over each panel, with the value proportional to the normal component of the velocity over that panel.

Both tangential and normal components of the velocity at each airfoil and jet control point are easily expressed as summations of completely known terms (influences of freestream and source distributions) and terms known except for a constant factor (influences of vortex distributions). These summations are substituted into the equations for the boundary conditions to produce a set of simultaneous algebraic equations for the unknown vortex strengths.

For purposes of verifying the suitability of the singularity representations and the accuracy of some of the numerical procedures, some conventional airfoil cases without jets have been solved. In these cases, the algebraic equations are all linear and extra provision for the trailing-edge Kutta condition has been made. Figures 3 and 4 show typical examples of the good agreement which has been obtained between the numerical and exact solutions for both thin and thick airfoil cases. Note particularly the good agreement near the flap hinge in Fig. 2, even though no special hinge singularity has been provided.

Jet Iteration Schemes

Perhaps the most obvious iteration scheme (and the one which is simplest to adapt from conventional potential flow computer programs) is the one used by Davenport,[6] Herold,[7] and initially by the present author. In this scheme, an initial jet shape is estimated (usually from the results of linear methods), the flow about the airfoil-jet

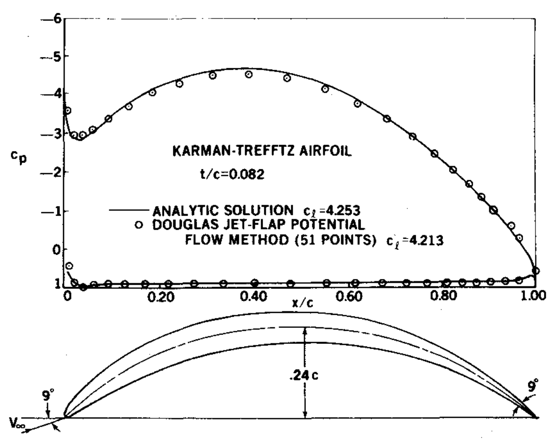

Fig. 4 Accuracy of the present method for a thick airfoil case.

combination is solved for (i.e., the kinematic boundary condition is satisfied), the resulting jet loading is used in conjunction with the dynamic boundary condition [Eq. (1)] to determine a new jet shape, and the process is repeated until convergence is obtained. Although in some cases this iterative scheme works quite well, strong damping of the changes in jet shape is almost always necessary and in many cases the iterations quickly diverge, as shown in Fig. 5.

A much more reliable iterative procedure is to solve the equations for the airfoil kinematic boundary condition and for the jet dynamic boundary condition, change the jet shape to lie along a streamline of the resulting flowfield, and repeat the process until convergence is obtained. Although the dynamic boundary condition equation is nonlinear, it can be approximated by linear equations in such a way that the errors become vanishingly small as the solution converges.

The dynamic boundary condition is expressed in terms of changes in flow angles rather than radii of curvature by integrating Eq. (2) along the jet.

$$2 \int (\gamma/V_\infty)(\bar{V}/V_\infty)\, ds = c_\mu \Delta\theta \tag{3}$$

where both the integral and the difference are taken between successive jet control points, except for the first jet equation in which case they are taken between the trailing edge and the first jet control point, and also the last equation in which case they are taken between the last jet control point and infinity.

Equation (3) is nonlinear for two reasons. The left-hand side is nonlinear because $\bar{V}$, the local flow velocity, is a function of the vortex distribution; the right-hand side becomes nonlinear when the flow angles are represented in terms of components of the velocity.

To make the left-hand side linear, $\bar{V}$ is assumed to be known each iteration, with its value given from the results of the previous iteration. It has been found that $\bar{V}$ almost always varies very slowly from one iteration to the next and, of course, as the iterations converge it stops changing at all. To simplify the integral, $\bar{V}$ is assumed to be constant over each straight line segment representing the jet. This is justified by noting that the results of solutions using the initial iterative scheme showed $\bar{V}$ to vary along the jet length at least one order more slowly than the vorticity, which is linear over each panel. If the airfoil and jet vortex sheets were completely planar, $\bar{V}$ would be exactly constant over the entire vortex sheet.

The right-hand side is made linear by approximating the flow angles on the jet by the sum of the angle of the assumed jet shape and the tangent of the angle of flow relative to it.

$$\theta \simeq \theta_{\text{assumed}} + V_n/\bar{V} \tag{4}$$

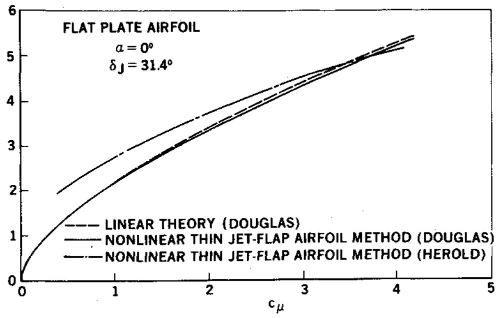

Fig. 6 Comparison of the present method with Herold's nonlinear thin jet-flapped airfoil method.

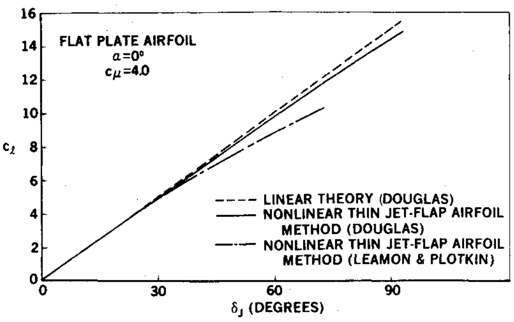

Fig. 7 Comparison of the present method with Leamon and Plotkin's nonlinear thin jet-flapped airfoil method.

where V_n is linear with respect to the vorticity and again $\bar{V}$ is known from the previous iteration. The difference in angle between successive jet control points is also linear with respect to the vorticity so Eq. (3) becomes completely linear.

$$2\bar{V}/V_\infty \int \gamma/V_\infty \, ds = c_\mu[\Delta\theta_{\text{assumed}} + \Delta(V_n/\bar{V})] \quad (5)$$

As the iterations proceed, V_n reduces to zero and the dynamic boundary condition approximations disappear.

Effects of Jet Entrainment

An approximate treatment of entrainment effects is included in the present method. A sink distribution is superimposed on the jet vortex sheet. The local sink strength is proportional to the local flow into the jet, which is assumed to be symmetrical above and below the jet. The local flow into the jet is determined, as suggested by Stratford,[11] by assuming it to be a constant factor of the difference between the jet speed and the external flow speed, both of which vary along the jet. The jet speed at the jet exit must be specified. The local jet speed is determined by requiring the axial increase in momentum of the jet to equal the loss of the tangential component of the momentum of the freestream. The local external flow speed is determined from the previous jet iteration. All other calculations and iterative procedures are unchanged.

Comparison with Other Methods

In general, there is not good agreement among the various nonlinear potential flow theories that are applicable to thin jet-flapped airfoils. The results of the two iterative schemes developed by the present author (which could be considered different methods) agree well with each other and with the results of Davenport's method,[6] for the limited amount of data available. However, Herold's method[7] and Leamon and Plotkin's method[9] apparently produce completely different results.

Figure 6 compares the results obtained by the present author and Herold[7] for a flat-plate airfoil with a jet deflection of 31.4°. This is a case where the linear theories agree well with experimental data (when corrected for thickness) and nonlinear effects ought to be fairly small. As expected the results of the present method are close to those of a linear method, but Herold's results are completely different. The trend he shows, nonlinear effects increasing with decreasing jet momentum, is completely unreasonable, since weaker jets lie more closely in one plane and perturb the flow velocities less than stronger jets.

Figure 7 shows the effect of jet deflection angle on the lift of a flat plate airfoil as calculated by linear theory, the present method, and Leamon and Plotkin's method.[9] As expected, all the results are the same at low jet deflection angles but, at larger angles, Leamon and Plotkin[9] show much larger nonlinear effects than the present method. For these cases, their method might be expected to be less accurate than for lower jet angles, because all parameters are given as functions of x (the streamwise coordinate) instead of s (the arc-length along the jet). As jet angles approach 90°, a small increment in x corresponds to a large increment in s, making definition of the parameters very difficult in the area near the trailing edge, where most of the jet curvature and vorticity are concentrated. The resulting jet curvature values given in Ref. 9, for the case with a jet deflection angle of 75°, are very irregular, indicating that perhaps their "improved" values are less accurate than the purely linear values.

It is not immediately apparent why the linear and nonlinear methods should agree so closely under conditions which clearly violate the assumptions of the linear theory, but this is consistent with the fact that Spence's method[2] has been used successfully for such cases before. The method of Ref. 6 apparently also agrees very closely with linear theory at large jet deflection angles, but very rew results are given. For a flat plate with a jet deflection angle of 90° and a momentum coefficient of 1.0 the results of the present method and Davenport's method[6] agree within about 1% and are both within about 5% of the linear result.

It should not be concluded from the above that linear theory is always adequate for analyzing jet-flapped airfoils. Many other cases show much larger discrepancies between linear and nonlinear theories, particularly cases with large mechanical flap deflections and cases in ground effect where the differences in lift can easily exceed 50%. For cases with no flaps, moderate camber, and out-of-ground effect, however, linear theory is remarkably accurate.

The only nonlinear thick-airfoil jet flap method available for comparison is the one developed by Sato.[8] Figure 8 compares the results of Sato's method and the present method for a NACA 0012 airfoil with jet and shows very good agreement except on the lower surface near the trailing edge. The differences in this region are probably due to the fact that the Kutta condition is satisfied at the

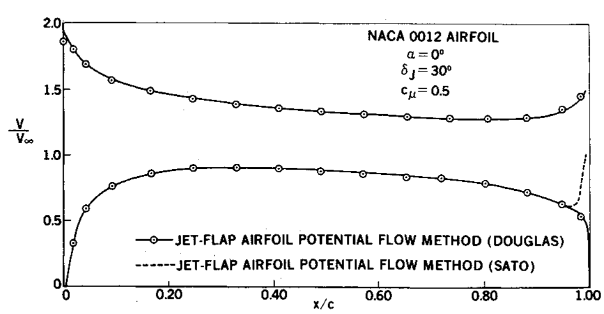

Fig. 8 Comparison of the present method with Sato's jet-flapped airfoil potential flow method.

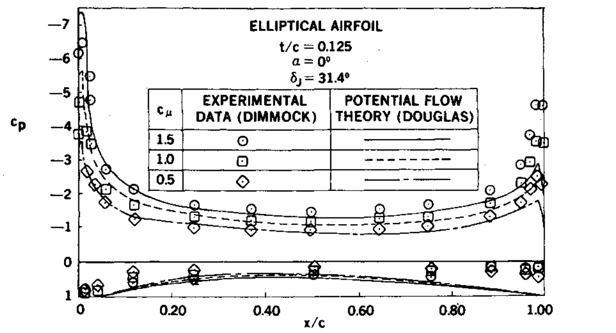

Fig. 9 Comparison of the present method with experimental data (neglecting jet entrainment effects).

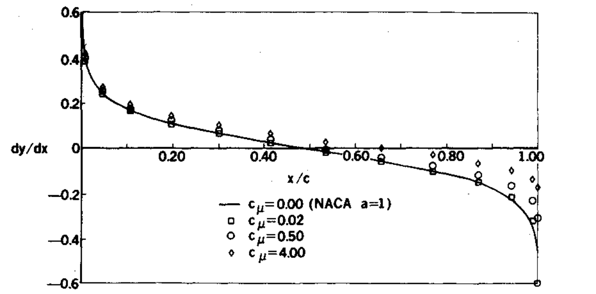

Fig. 11 Effect of jet momentum on the camber required for constant vorticity (γ/V_∞ = 0.5).

trailing edge in Sato's method, rather than at infinity, so that the dividing streamline leaves the airfoil along the bisector of the trailing edge angle, rather than at the jet deflection angle.

Comparison with Experimental Data

The present method has been used to analyze the elliptical airfoil tested by Dimmock.[12] Figure 9, showing the pressure distributions for three different values of the jet momentum coefficient, indicates large discrepancies between the experimental and theoretical pressure, especially on the lower surface and near the trailing edge on the upper surface. Figure 10 shows that these discrepancies disappear when entrainment effects are included in the calculations.

Solution of the Jet Flap Problem—Airfoil Design Methods

The design of an airfoil to produce a specified velocity distribution is considerably more difficult than the analysis of an airfoil of given shape. Consequently, there are many more airfoil analysis methods than design methods in existence. The only jet-flapped airfoil design method known to the present author is the one described below.

In contrast to the author's analysis method, which is applicable to numerous airfoil configurations and flow conditions and which is believed to be very nearly exact, the design method is applicable only to mono-element jet-flapped airfoils and the numerical procedures are more approximate. It is very similar in several respects to the early conventional airfoil design methods described in Ref. 13 and elsewhere, in which camber and thickness distributions are designed separately.

More specifically, the following steps are taken:

1) Specify the desired velocity distribution and jet momentum coefficient.

2) Divide the velocity distribution into symmetric and antisymmetric portions.

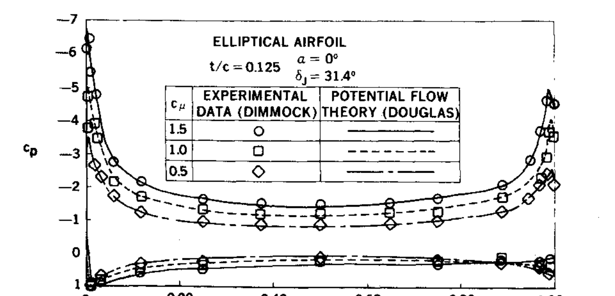

Fig. 10 Comparison of the present method with experimental data (including jet entrainment effects).

3) Design the camber distribution needed to produce the antisymmetric portion of the velocity distribution (vorticity).

4) Design the thickness distribution needed to produce the symmetrical portion of the velocity distribution.

5) Combine camber and thickness distributions.

6) Using the nonlinear analysis method described earlier, determine how large the discrepancies are between the specified and resulting velocity distributions and, if necessary, modify the shape.

Camber Design

Linear assumptions are made in the present method for designing the camberline needed to produce a specified airfoil vorticity distribution. The mathematical problem is to determine the jet vorticity needed to satisfy the jet boundary conditions in the presence of the downwash field induced by the known airfoil vorticity. Once the jet vorticity is known, the upwash field on the wing (and hence camber distribution) due to the airfoil and jet vorticity is easily determined.

As in the airfoil analysis method, the jet boundary conditions are given by Eq. (5). However, since linearized assumptions are made, $\bar{V}/V_\infty$ is equal to unity, $\Delta\theta_{\text{assumed}}$ is zero, and V_n is simply the downwash on the jet. The downwash consists of the known contribution of the airfoil vorticity and the unknown contribution of the jet vorticity. The resulting equations are all linear with respect to the jet vorticity and can be solved without iterating.

In order to avoid over-specifying the problem, either the initial jet deflection angle or the value of the airfoil vorticity at the trailing edge must be considered to be unknown.

The first option has been used to design a family of camberlines having constant vorticity distributions on the airfoil, for several different values of the jet momentum coefficient. All the resulting camberlines of this family have the jet issuing tangentially to the trailing edge. This is a result of requiring a finite vorticity at the trailing edge; there can be no corner there. In the limit as the jet momentum approaches zero, these camberlines should become indistinguishable from the NACA a = 1 camberline, except that, in the present case, the vorticity is required to be zero at the leading edge and is only constant aft of about the first half percent of the chord. This removes the logarithmic singularity in the camber distribution at the leading edge. The effect of the jet momentum is to reduce the amount of camber required to obtain a constant vorticity distribution, especially near the trailing edge (Fig. 11). Since the NACA a = 1 camberline has very large camber near the trailing edge, nonlinear effects and viscous effects might be expected to prevent the desired flow from actually being obtained. The camberlines designed to be used in conjunction with jet flaps have more modest aft-camber and, therefore, the linearized design procedure might be expected to be more realistic. Figure 12 shows

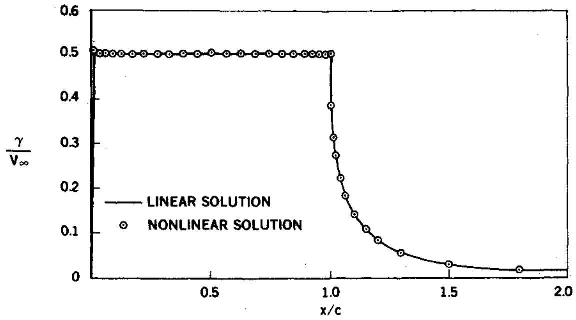

Fig. 12 Accuracy of the linear solution for a camberline designed for constant vorticity (c_μ = 0.5).

that at least the nonlinear effects are very small for a typical case.

In designing a camberline to be combined with a particular thickness distribution, it may be more useful to employ the other camber design option (specifying the jet deflection angle relative to the chordline and leaving the trailing-edge vorticity free). In this way an unwanted corner in the flow can sometimes be avoided.

Thickness Design

Under the thin-jet assumptions made throughout this paper, the design of a jet-flapped airfoil's thickness distribution is no different than the design of an unpowered symmetrical airfoil. Therefore, any conventional airfoil method can be used for this portion of the design procedure.

Jet Entrainment Effects

The approximate influence of jet entrainment is considered by determining the influence of a sink distribution, lying along the semi-infinite line representing the jet, on the velocity distribution at the (linearized) camberline, and subtracting this velocity from that which the thickness distribution is required to contribute. The sink strength is determined under the assumption that the local flow velocity is equal to the freestream value.

Results

The initial tests of this design procedure were made using velocity distributions which were chosen in such a way that the symmetrical portion corresponded to the flow about known symmetrical airfoil sections. Figure 13 shows an airfoil having a NACA 65-010 thickness distribution and a camberline designed for constant loading with a momentum coefficient of 0.5. Figure 14 shows three airfoils, each having a NACA 63A-010 thickness distribution and a camberline designed to produce nearly constant upper-surface velocities.

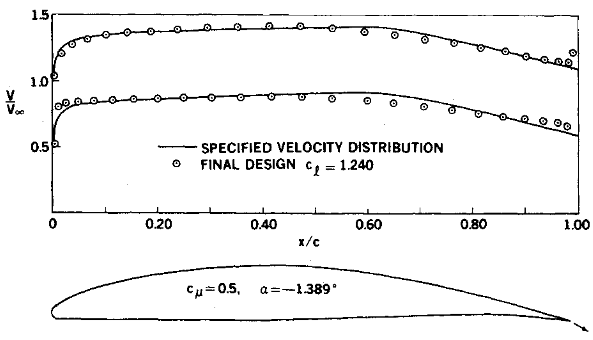

Fig. 13 Airfoil designed using a constant-vorticity camberline.

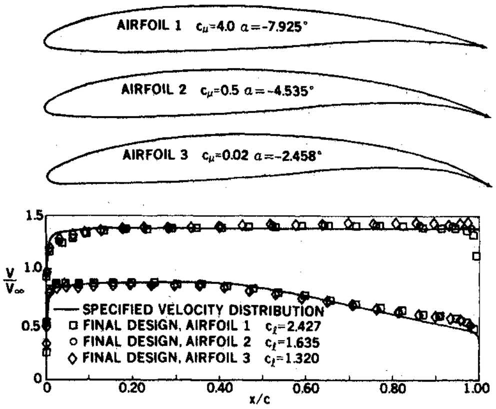

Fig. 14 Airfoils designed to have nearly constant upper-surface velocities.

Using the jet-flapped airfoil design method and including the effects of jet entrainment, it is possible to design airfoils having no adverse velocity gradients on either upper or lower surfaces. One such airfoil is shown in Fig. 15. This airfoil should have exceptionally good boundary-layer characteristics and very low drag. However, it is not necessarily the airfoil having the maximum lift-to-drag ratio for the same values of jet velocity, jet momentum coefficient and lift coefficient, because of the effect of jet entrainment on the airfoil drag. The optimum velocity distribution including this effect has not yet been determined.

In most of the cases shown in Figs. 13–15, the agreement between the specified velocity distributions and the values calculated using the nonlinear analysis method is very good. Under conditions of very high lift, the small-perturbation assumptions on which the method is based break down. In these cases, simple nonlinear corrections can be applied to improve the results. The main nonlinear correction accounts for the effect of the camber and jet on the symmetrical portion of the velocity distribution. To do this, the camber distribution is analyzed using the nonlinear analysis method and the resulting symmetrical portion of the velocity distribution is subtracted from the velocity which the thickness distribution must contribute. Note that this is very similar to the method of treating jet entrainment effects.

Other smaller improvements are made by slight modifications to the jet deflection angle and by assuming the jet momentum coefficient to be nondimensionalized using the total length of the camberline, rather than the straightline distance from leading to trailing edges.

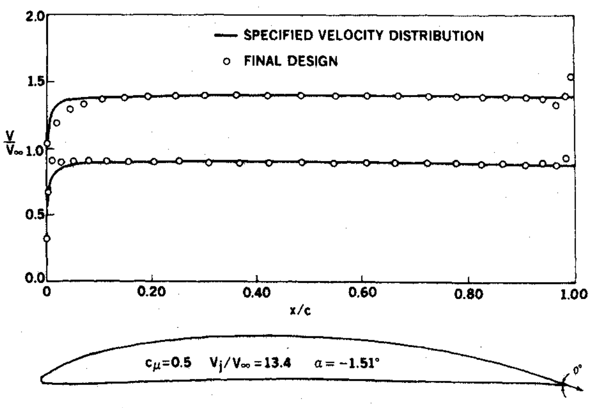

Fig. 15 Airfoil designed to have nearly constant upper- and lower-surface velocities (including jet entrainment effects).

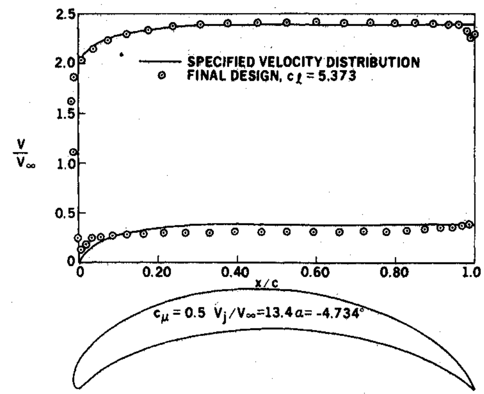

Fig. 16 High-lift airfoil designed to have nearly constant upper- and lower-surface velocities (including jet entrainment effects).

A high-lift airfoil having only very small regions of adverse velocity gradient is shown in Fig. 16. With the above modifications, the specified and final velocity distributions agree nearly as well as in the low-lift cases.

Conclusions

Methods have been developed for analyzing the flow about jet-flapped airfoils of given shape and for determining the shape required to produce a given velocity distribution. The analysis method is capable of treating a wide variety of flow problems involving jet-flapped airfoils. Comparisons with available solutions and experimental data indicate that the results of the analysis method are very good. The airfoil design method is more approximate, but analysis of the resulting airfoils indicates that good accuracy can be obtained for many cases.

The analysis method is essentially complete, insofar as it is applied to internally-ducted jet-flapped airfoils in incompressible flow. Recent projects at Douglas have involved the consideration of compressibility effects and extension of the method to cases for which jet thickness is important, such as the externally blown flap systems.

The design method, as described above, is still in an early stage of development. Work has begun on a second-order nonlinear method and a fully nonlinear method may soon be developed.

Much scope remains for two-dimensional jet flap experimental work to provide further verification of the accuracy of the analysis method and also to determine whether the airfoils resulting from the design method can retain their favorable characteristics when the practical problems of ducting, structure, and manufacturing are considered.

References

[1]Hagedorn, H. and Ruden, P., "Windkanaluntersuchunger an Elnem Junkers-Doppelflügel Mit Ausblaseschlits am Heck Des Hauptflügels," R.A.E. Translation 442, Dec. 1953, Royal Aircraft Establishment, Farnborough, England.

[2]Spence, D. A., "The Lift Coefficient of a Thin Jet-Flapped Wing," *Proceedings of the Royal Society,* Ser. A, Vol. 238, No. 1212, Dec. 1956, pp. 46–68.

[3]Shen, C. C., Lopez, M. L., and Wasson, N. F., "A Jet-Wing Lifting Surface Theory Using Elementary Vortex Distributions," AIAA Paper 73-652, Palm Springs, Calif., 1973.

[4]Hess, J. L. and Smith, A. M. O., "Calculation of Potential Flow about Arbitrary Bodies," *Progress in Aeronautical Science Series,* Vol. 8, Pergamon Press, New York, 1966.

[5]Stevens, W. A., Goradia, S. H., and Braden, J. A., "Mathematical Model for Two-Dimensional Multi-Component Airfoils in Viscous Flow," CR-1843, 1971, NASA.

[6]Davenport, F. J., "Singularity Solutions to General Potential Flow Airfoil Problems," Rep. D6-7202, 1962, Boeing Co., Seattle, Wash.

[7]Herold, A. C., "A Two-Dimensional, Iterative Solution for the Jet Flap," CR-2190, 1973, NASA.

[8]Sato, J., "Discrete Vortex Method of Two-Dimensional Jet Flaps," *AIAA Journal,* Vol. 11, No. 7, July 1973, pp. 968–973.

[9]Leamon, R. G. and Plotkin, A., "An Improved Solution of the Two-Dimensional Jet-Flapped Airfoil Problem," *Journal of Aircraft,* Vol. 9, No. 9, Sept. 1972, pp. 631–635.

[10]Keune, F., "Lift on a Bent Flat Plate," TM 1340, 1936, NACA.

[11]Stratford, B. S., "Mixing and the Jet Flap," *Aeronautical Quarterly,* Vol. VII, May 1956, pp. 85–105.

[12]Dimmock, N. A., "An Experimental Introduction to the Jet Flap," CP344, Oct. 1957, Aeroneutical Research Council, London, England.

[13]Abbott, I. H. and Von Doenhoff, A. E., *Theory of Wing Sections,* Dover, New York, 1959.